Konstantinos Spiliopoulos (Boston University)

Title: Stochastic gradient descent in continuous time and deep learning for PDEs.

Abstract: Stochastic gradient descent in continuous time (SGDCT) provides a computationally efficient method for the statistical learning of continuous-time models, which are widely used in science, engineering, and finance. The SGDCT algorithm follows a (noisy) descent direction along a continuous stream of data. SGDCT performs an online parameter update in continuous time, with the parameter updates satisfying a stochastic differential equation. We prove almost sure convergence of the algorithm and also establish a central limit theorem, characterizing the rate of convergence, for objective functions that can be non-convex. The convergence proofs leverages ergodicity by using an appropriate Poisson equation to help describe the evolution of the parameters for large times. SGDCT can also be used to solve continuous-time optimization problems, such as American options for financial applications. For certain continuous-time problems, SGDCT has some promising advantages compared to a traditional stochastic gradient descent algorithm. We combine SGDCT with a deep learning algorithm similar in spirit to Galerkin methods (called "Deep Galerkin Method (DGM)"), using a deep neural network instead of linear combinations of basis functions. As an example application, SGDCT is combined with DGM to price high-dimensional American options (up to 200 dimensions), which are free boundary problems, and Hamilton Jocobi-Bellman equations.