6th Summer School

MATH@NTUA

MATHEMATICAL THEORY OF INVERSE PROBLEMS AND APPLICATIONS

ATHENS, 26-30 JUNE 2023

Program

Monday, June 26th

10:00 – 11:30	Andreas Kirsch (Karlsruhe Institute of Technology, Germany) **An Introduction to Inverse Problems (Part I)**
11:30 – 12:00	*Coffee Break*
12:00 – 13:30	Andreas Kirsch (Karlsruhe Institute of Technology, Germany) **An Introduction to Inverse Problems (Part II)**
13:30 – 15:00	*Lunch Break*
15:00 – 16:00	Leonidas Mindrinos (Agricultural University of Athens, Greece) **A dimension-reduction method for the numerical solution of various Cauchy problems in R^2**

Tuesday, June 27th

10:00 – 11:30	Fioralba Cakoni (Rutgers University, USA) **The transmission eigenvalue problem, non-scattering phenomena and inverse scattering for inhomogeneous media (Part I)**
11:30 – 12:00	*Coffee Break*
12:00 – 13:30	Fioralba Cakoni (Rutgers University, USA) **The transmission eigenvalue problem, non-scattering phenomena and inverse scattering for inhomogeneous media (Part II)**
13:30 – 15:00	*Lunch Break*
15:00 – 16:00	Antonios Charalambopoulos (NTUA, Greece) **On detecting and disclosing hidden acoustic inhomogeneities, by stochastically following random, though monitored, walks**

Wednesday, June 28th

10:00 – 11:30	Marc Bonnet (ENSTA Paris, France) **The concept of topological derivative in qualitative inverse scattering (Part I)**
11:30 – 12:00	*Coffee Break*
12:00 – 13:30	Marc Bonnet (ENSTA Paris, France) **The concept of topological derivative in qualitative inverse scattering (Part II)**
13:30 – 15:00	*Lunch Break*

Thursday, June 29th

10:00 – 11:30	Mourad Sini (Radon Institute, Austria) **Inverse Problems in Imaging using Resonant Contrast Agents (Part I)**
11:30 – 12:00	*Coffee Break*
12:00 – 13:30	Mourad Sini (Radon Institute, Austria) **Inverse Problems in Imaging using Resonant Contrast Agents (Part II)**
13:30 – 15:00	*Lunch Break*
15:00 – 16:00	Drossos Gintides (NTUA, Greece) **Uniqueness for recovering the refractive index from far field data or from the knowledge of transmission eigenvalues**

Friday, June 30th

10:00 – 11:30	Guillaume Bal (University of Chicago, USA) **Mathematical aspects of hybrid inverse problems (Part I)**
11:30 – 12:00	*Coffee Break*
12:00 – 13:30	Guillaume Bal (University of Chicago, USA) **Mathematical aspects of hybrid inverse problems (Part II)**
13:30 – 15:00	*Lunch Break*

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Book of Abstracts

A) Invited Speakers (in chronological order)

An Introduction to Inverse Problems

Andreas Kirsch, Karlsruhe Institute of Technology (KIT)

In this series of talks a very first introduction into the mathematical theory of inverse problem is presented. We begin with some classical examples, such as the inverse heat equation, the problem of impedance tomography, and an inverse scattering problem.

In the second part we sketch the idea of regularization and restrict ourselves to Tikhonov's regularization method and Landweber's method as an example of an iterative regularization scheme. We finish the second part with some comments on other regularization techniques. In the third part we consider the direct and inverse scattering problems for the scattering of time-harmonic plane waves by some inhomogeneous medium. We sketch an integral equation approach for the treatment of the direct problem, introduce the far-field pattern, and present the idea of the Factorization Method for the determination of the support of the contrast.

The transmission eigenvalue problem, non-scattering phenomena and inverse scattering for inhomogeneous media

Fioralba Cakoni, Rutgers University

This lecture is a continuation of the lecture given by Prof. Andreas Kirsch. The focus of this lecture is the role of the transmission eigenvalue problem for solving the inverse scattering problem for inhomogeneous media.

The transmission eigenvalues are related to non-scattering wave numbers for which there exists an incident wave that does not scatterer by the given medium, in other words the far field operator (or relative scattering operator) is not injective with not dense range. The transmission eigenvalue problem has a deceptively simple formulation, namely, two elliptic PDEs in a bounded domain (one governs the wave propagation in the scattering medium and the other in the background that occupies the support of the medium) that share the same Cauchy data on the boundary but presents a perplexing mathematical structure. In particular, it is a non-self-adjoint eigenvalue problem for a non-strongly elliptic operator, and hence the investigation of its spectral properties becomes challenging. Roughly, the spectral properties depend on the assumptions on the contrasts in the media near the boundary.

We will discuss questions central to the inverse scattering theory include discreteness of the spectrum that is closely related to the determination of the support of inhomogeneity from scattering data using linear sampling and factorization methods, location of transmission eigenvalues in the complex plane that is essential to the development of the time domain linear sampling method , and the existence of transmission eigenvalues as well as the accurate determination of real transmission eigenvalues from scattering data, which has become important since real transmission eigenvalues could be used to obtain information about the scattering media. We will conclude with some recent developments related necessary conditions on the regularity of inhomogeneity for a transmission eigenvalue to be non-scattering wave number.

The concept of topological derivative in qualitative inverse scattering

Marc Bonnet, ENSTA Paris

In this lecture, we address qualitative inverse scattering approaches based on asymptotic approximations of the scattering of waves by inhomogeneities of small size. We show how such models can be obtained and justified by means of volume integral equation (VIE) approaches, and then incorporate them into imaging functionals based on the concept of topological derivative. We discuss mathematical arguments and results that support TD-based imaging, and present numerical experiments. We finally describe extensions of the foregoing approaches that exploit asymptotic models (i) of higher order or (ii) for surface-breaking defects.

Inverse Problems in Imaging using Resonant Contrast Agents

Mourad Sini, Radon Institute

Many of the traditional inverse problems use remotely measured responses to wave-based interrogations in order to reconstruct (few) material coefficients interring into the considered model. Such inverse problems are known to be highly unstable. In several imaging modalities, it is advised, in the engineering community, to first inject contrast agents into the domain of interests before performing the experiments. With such perturbations, one can enhance the contrast appearing in the domain and hence 'see better'.

In these lectures, we propose an approach on how to model and understand such contrast agents. Most importantly, we show, by examples of imaging modalities, how the generated contrast can be used to extract the needed material coefficients. We divide the 3-hours lecture into 3 lectures of 'one hour' each.

1. In the first hour, we provide with a classification of these contrast agents as subwavelenght resonators. We discuss two types of them, namely the acoustic and electromagnetic resonators. We give systematic links between their contrasting properties and the possible resonant frequencies they can generate.
2. In the second hour, we consider the imaging modality using ultrasound waves in the time-harmonic regime. For this case, the measured data are the back-scattered waves in a <u>single</u> direction with a large enough <u>band of incident frequencies</u>, repeated by injecting in the domain of interest appropriate acoustic bubbles. With such measurements, we first extract the acoustic resonance frequencies (like the Minnaert one) generated by the bubbles from which we can recover the mass density. Second, we also extract the internal values of the acoustic waves from which we recover the bulk modulus.
3. In the third hour, we move on to the imaging modality using ultrasound waves in the time-domain regime. In this case, we measure the acoustic wave at a <u>single point</u>, outside the domain of imaging Ω, in a <u>large enough time interval</u>. Such experiments are repeated by injecting acoustic bubbles in Ω. From such data, we first extract the internal values of the travel time function and then we derive the acoustic speed of propagation from the Eikonal equation. Second, we also extract the internal values of the total acoustic wave from which we recover the bulk modulus. From these two coefficients, we deduce the values of the mass density.

The mass density and the bulk modulus are the two main acoustic coefficients modeling the ultrasound waves.

These lectures are based on the following material:

- Dabrowski, A. Ghandriche, M. Sini, Mathematical analysis of the acoustic imaging modality using bubbles as contrast agents at nearly resonating frequencies, Inverse Probl. Imaging, 15 (2021), 555–597.
- A. Ghandriche, M. Sini, Photo-acoustic inversion using plasmonic contrast agents: the full Maxwell model. J. Differential Equations 341 (2022), 1–78.
- S. Senapati, M. Sini, H. Wang, Recovering both the wave speed and the source function in a time-domain wave equation by injecting high contrast bubbles. arXiv:2304.08869.
- A. Ghandriche, M. Sini, Simultaneous Reconstruction of Optical and Acoustical Properties in Photo-Acoustic Imaging using plasmonics. arXiv:2209.08482

Mathematical aspects of hybrid inverse problems

Guillaume Bal, University of Chicago

These lectures provide an introduction to hybrid, or coupled-physics, medical imaging modalities that aim to combine high contrast with high resolution. We focus on the modalities called photo-acoustic tomography and elastography. We describe the different inverse problems associated with these imaging methods and focus on their injectivity and stability properties.

B) Local Speakers (in chronological order)

A dimension-reduction method for the numerical solution of various Cauchy problems in R^2

Leonidas Mindrinos, Agricultural University of Athens

In this lecture we will discuss a numerical method for solving initial boundary value problems in two dimensions. The method consists of two parts: In the first step we consider a time-discretization by applying a weighted Laguerre transformation on the problem. This technique reduces the time-dependent problem to a sequence of stationary problems for the spatial-dependent Fourier-Laguerre coefficients of the series representation. In the second step, we reduce the 2D problems to a system of integral equations with the help of well-known boundary integral equation method. In most cases, a single-layer representation of the coefficients is sufficient. We solve the discretized and linearized problem on the boundary with the collocation method. We handle the singularities with quadrature rules.

This lecture is based on the joint works with R. Chapko (Ivan Franko University of Lviv, Ukraine) and B. T. Johansson (Linköping University, Sweden).

On detecting and disclosing hidden acoustic inhomogeneities, by stochastically following random, though monitored, walks

Antonios Charalambopoulos, NTUA

The current course refers to the presentation of a novel stochastic method in order to face the time-reduced inverse scattering problem, governed by Helmholtz equation, outside connected or disconnected obstacles supporting boundary conditions of Dirichlet type.

On the basis of the stochastic analysis, a series of efficient and alternative stochastic representations of the scattering field have been constructed. These novel representations constitute conceptually the probabilistic analogue of the well-known deterministic integral representations involving the famous Green's functions, and so merit special importance. Their advantage lies on their intrinsic probabilistic nature, allowing the solution of the direct and inverse scattering problem in the realm of local methods, which are strongly preferable in comparison with the traditional global ones.

The aforementioned locality reflects the ability to handle the scattering field only in small bounded portions of the scattering medium, by monitoring suitable stochastic processes, confined in narrow sub-regions where data are available. Especially in the realm of the inverse scattering problem, two different schemes are proposed facing reconstruction from the far field and near field data, respectively. The crucial characteristic of the inversion is that the reconstruction is fulfilled through stochastic experiments, taking place in the interior of conical regions whose base belong to the data region, while their vertices detect appropriately the supporting surfaces of the sought scatterers.

Uniqueness for recovering the refractive index from far field data or from the knowledge of transmission eigenvalues

Drossos Gintides, NTUA

This lecture is related to the lectures given by Prof. Andreas Kirsch and Prof. Fioralba Cakoni. In inverse scattering theory we want to determine the refractive index of an inhomogeneity from far field data. We will discuss two different ways of uniquely determining the refractive index.

The first approach is directly related to the knowledge of the far field data for many or few incident plane waves. We will review the uniqueness theorem due to A. Nachmann, R. Novikov and A. G. Ramm in R^3 based on the complete knowledge of the far fields for infinitely many incident plane waves. Next, we will present a nice uniqueness estimate for only one incident plane wave and the specific geometry of a disk in R^2 with constant refractive index using separation of variables and the properties of the transmission eigenvalue problem. This result is due to R. Kress and A. Altundag.

The second approach is based on the complete or partial knowledge of the set of transmission eigenvalues. This is an inverse spectral problem and in the literature is called the inverse transmission eigenvalue problem. We will discuss results for the determination of a spherically stratified index of refraction, that is the refractive index is a function n(r), where r is the radial distance. This problem was considered initially by J. McLaughlin and P. Polyakov and J. McLaughlin, P. Polyakov and P. Sachs, who showed that n(r) is uniquely determined only if certain restrictions are made on the unknown function and under the assumption that the corresponding eigenfunctions are spherically symmetric. We will also discuss a uniqueness result, from a joint work with F. Cakoni and D. Colton, which states that the complete set of transmission eigenvalues uniquely determines the refractive index n(r) under the assumption that n(r) is C^2 and n(0) is known. Finally, we will refer to extensions when discontinuities are present from a joint work with N. Pallikarakis.